Mathematical Logic

SEC-I

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Logic

- Mathematical proofs, or more specifically, mathematical arguments depend on reasoning.

- From a set of axioms, or a set of assumptions, a mathematician applies reasoning to argue systematically and study the validity of arguments, consistency of situations and arrive at a decision regarding the truth of some statement.

- Logic is the system of tool of this reasoning.

Deductions

- Logic is all about deducing an inference by using reasoning, and equivalently, checking the truth/falsehood of a statement.

- A statement (or system of statements) is held logically true if it is true when the premises logically lead to them and if there is no inherent inconsistency in them.

Propositions

- In Mathematics, one deals with statements/assertions that can be **either true or false** (but not both, or not ambiguous). Such statements are called **propositions**.

- There might be ambiguous statements or statements which do not come under the category of strictly true/ false in literature, but mathematical logic is not applicable there.

Examples and non-examples of propositions

- **The following are all propositions**:

(i) Logic and Sets is a course in SEC in SEM-III.
(ii) RSG teaches the Logic course in BNMV.
(iii) 20 students have taken the Logic elective in BNMV.

- **However, the following statements, exclamatory or imperative in nature, cannot be judged for truth/falsehood, and hence are not propositions**:

(i) What a wonderful shot by Sachin !
(ii) Go and bring a bottle of water please.

Simple and complex propositions

- The propositions stated so far are simple/primitive in nature in the sense that they cannot be divided into simpler propositions. However, we can come across propositions such as the following:

(i) He is not doing what is good for him.

(ii) if it rains, I will not go out.

(iii) The cyclist hit the man and drove on.

(iv) I will go to Darjeeling or Puri.

(v) One gets the prize if and only if one scores 90%.

Each of these can be further resolved into propositions and hence they are called complex propositions.

Logical connectives

- Complex propositions are formed by applying logical connectives to simple propositions. These logical connectives are derived from natural language, but act a little differently in certain respects (although the purpose is more or less the same).

Logical connectives

- The most frequently used logical connectives are

(i) And (Conjunction)

(ii) Or (Disjunction)

(iii) if… then (Implication)

(iv) not (negation)

(v) if and only if (Biconditional)

Truth table for And

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth table for Or

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Truth table for if, then

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Equivalent truth table for if, then

p	q	~p	~p ∨ q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Truth table for Not

p	~p
T	F
F	T

Truth table for if and only if

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth table for nor

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

Truth table for XOR

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

Truth table for Logically true/false

p	T
T	T
F	T

or

p	T
1	1
0	1

p	F
T	F
F	F

or

p	F
1	0
0	0